

**Problem T-1**

Let  $\mathbb{Z}$  denote the set of all integers and  $\mathbb{Z}_{>0}$  denote the set of all positive integers.

- (a) A function  $f: \mathbb{Z} \rightarrow \mathbb{Z}$  is called  $\mathbb{Z}$ -good if it satisfies  $f(a^2 + b) = f(b^2 + a)$  for all  $a, b \in \mathbb{Z}$ . Determine the largest possible number of distinct values that can occur among  $f(1), f(2), \dots, f(2023)$ , where  $f$  is a  $\mathbb{Z}$ -good function.
- (b) A function  $f: \mathbb{Z}_{>0} \rightarrow \mathbb{Z}_{>0}$  is called  $\mathbb{Z}_{>0}$ -good if it satisfies  $f(a^2 + b) = f(b^2 + a)$  for all  $a, b \in \mathbb{Z}_{>0}$ . Determine the largest possible number of distinct values that can occur among  $f(1), f(2), \dots, f(2023)$ , where  $f$  is a  $\mathbb{Z}_{>0}$ -good function.

**Problem T-2**

Let  $a, b, c$  and  $d$  be positive real numbers with  $abcd = 1$ . Prove that

$$\frac{ab+1}{a+1} + \frac{bc+1}{b+1} + \frac{cd+1}{c+1} + \frac{da+1}{d+1} \geq 4,$$

and determine all quadruples  $(a, b, c, d)$  for which equality holds.

**Problem T-3**

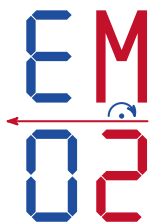
Find the smallest integer  $b$  with the following property: For each way of colouring exactly  $b$  squares of an  $8 \times 8$  chessboard green, one can place 7 bishops on 7 green squares so that no two bishops attack each other.

*Remark.* Two bishops attack each other if they are on the same diagonal.

**Problem T-4**

Let  $c \geq 4$  be an even integer. In some football league, each team has a home uniform and an away uniform. Every home uniform is coloured in two different colours, and every away uniform is coloured in one colour. A team's away uniform cannot be coloured in one of the colours from the home uniform. There are at most  $c$  distinct colours on all of the uniforms. If two teams have the same two colours on their home uniforms, then they have different colours on their away uniforms.

We say a pair of uniforms is *clashing* if some colour appears on both of them. Suppose that for every team  $X$  in the league, there is no team  $Y$  in the league such that the home uniform of  $X$  is clashing with both uniforms of  $Y$ . Determine the maximum possible number of teams in the league.



**Problem T–5**

We are given a convex quadrilateral  $ABCD$  whose angles are not right. Assume there are points  $P, Q, R, S$  on its sides  $AB, BC, CD, DA$ , respectively, such that  $PS \parallel BD$ ,  $SQ \perp BC$ ,  $PR \perp CD$ . Furthermore, assume that the lines  $PR, SQ$ , and  $AC$  are concurrent. Prove that the points  $P, Q, R, S$  are concyclic.

**Problem T–6**

Let  $ABC$  be an acute triangle with  $AB < AC$ . Let  $J$  be the center of the  $A$ -excircle of  $ABC$ . Let  $D$  be the projection of  $J$  on line  $BC$ . The internal bisectors of angles  $BDJ$  and  $JDC$  intersect lines  $BJ$  and  $JC$  at  $X$  and  $Y$ , respectively. Segments  $XY$  and  $JD$  intersect at  $P$ . Let  $Q$  be the projection of  $A$  on line  $BC$ . Prove that the internal angle bisector of  $QAP$  is perpendicular to line  $XY$ .

*Remark.* The  $A$ -excircle of the triangle  $ABC$  is the circle outside the triangle which is tangent to the lines  $AB, AC$ , and the line segment  $BC$ .

**Problem T–7**

Find all positive integers  $n$  for which there exist positive integers  $a > b$  satisfying

$$n = \frac{4ab}{a-b}.$$

**Problem T–8**

Let  $A$  and  $B$  be positive integers. Consider a sequence of positive integers  $(x_n)_{n \geq 1}$  such that

$$x_{n+1} = A \cdot \gcd(x_n, x_{n-1}) + B \quad \text{for every } n \geq 2.$$

Prove that the sequence attains only finitely many different values.

*Remark.* We denote by  $\gcd(a, b)$  the greatest common divisor of positive integers  $a$  and  $b$ .